

THE STRING PHASES OF HAWKING RADIATION, DE SITTER STAGE AND DE BROGLIE TYPE DUALITY

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Abstract

We explicitly describe the last stages of black hole evaporation in the context of string theory : the combined study of Quantum Field Theory (QFT) and String Theory (ST) in curved backgrounds allows us to go further in the understanding of quantum gravity effects. The string “analogue model”(or thermo-dynamical approach) is a well suited framework for this purpose. The results also apply to another physically relevant case: de Sitter background. Semiclassical (QFT) and quantum gravity (String) phases or regimes are properly determined (back reaction effects included). The Hawking-Gibbons temperature T_H of the semiclassical regime becomes the intrinsic string temperature T_S in the quantum gravity regime. The spectrum of black hole evaporation is an incomplete gamma function of $(T_S - T_H)$: the early evaporation is thermal (Hawking radiation), while at the end the black hole undergoes a phase transition to a string state decaying (as string decay) into pure (non mixed) particle states. Remarkably, explicit dynamical computations show that both gravity regimes: semiclassical (QFT) and quantum (string), are dual of each other, in the precise sense of the classical-quantum (de Broglie type) duality.

1 Introduction

The combined study of Quantum Field Theory (QFT) and Quantum String Theory (QST) in curved space times - and in the framework of the string analogue model - allows to go further in the understanding of quantum gravity effects Ref. [1,2,3,4,16,17].

The string “analogue model” (or thermodynamical approach) is a suitable framework for this purpose. Strings are considered as a collection of quantum fields ϕ_n which do not interact among themselves but are coupled to the curved background, and whose masses are given by the degenerated string spectrum in this background. The higher mass spectrum is described by the density of string mass levels $\rho(m)$ in the space time considered Ref. [3,4].

In Black Hole backgrounds (BH), $\rho(m)$ is the same as in flat space time and the string critical temperature is the usual (Hagedorn) temperature of strings in flat space time Ref. [5]. In de Sitter (dS) as well as in Anti de Sitter (AdS) backgrounds, (including the corresponding conformal invariant WZWN SL(2,R) models), $\rho(m)$ are different from flat space time Ref. [3,6,7,8,9,17]. There is a critical (maximal) string temperature in de Sitter background, while in AdS there is no finite maximal string temperature at all. Ref. [6,7,9] (the ”maximal” temperature is formally infinite, as the partition function for a gas of strings in AdS is defined for any external temperature Ref. [6]).

The explicit results of quantum string dynamics in curved backgrounds developed in refs [3], [4],[16],[17] lead to a \mathcal{R} “dual” (classical-quantum) transformation which, in particular, over a length L , simply reads:

$$\tilde{L} = \mathcal{R}L = \mathcal{L}_{\mathcal{R}} L^{-1} \quad (1)$$

where $\mathcal{L}_{\mathcal{R}}$ has dimensions of (length)². For physical processes, \mathcal{R} maps classical length scales L_{cl} into quantum string length scales L_q , and conversely:

$$\tilde{L}_{cl} \equiv \mathcal{R}L_{cl} = \mathcal{L}_{\mathcal{R}} L_{cl}^{-1} = L_q \quad (2)$$

and

$$\tilde{L}_q \equiv \mathcal{R}L_q = \mathcal{L}_{\mathcal{R}} L_q^{-1} = L_{cl} \quad (3)$$

$\mathcal{L}_{\mathcal{R}}$ depends on the dimensional parameters of the theory and on the string constant α' (and on \hbar , c as well). The \mathcal{R} transform is not an assumed or *a priori* imposed symmetry, but it is a transformation *revealed* from the QFT and QST explicit dynamical calculations in curved backgrounds Ref. [1,2,3,4,16,17].

L_{cl} sets up a length scale for the semiclassical - QFT regime and L_q is the length scale that characterizes the string domain; L_q depends on the dimensional string constant α' ($\alpha' = c^2/2\pi T$, where T is the string tension) and on the specific background considered. The ”vehicle” of the \mathcal{R} -transformation is the dimensional constant α' of string theory (ST). The \mathcal{R} -operation transforms the characteristic lengths of one regime (QFT or QST) into the characteristic lengths of the other, and thus relate different physical regimes in the *same* curved background.

2 Classical-Quantum Duality Relations

The \mathcal{R} -relation above described *does not needs a priori* any symmetry or compactified dimensions. It *does not* require the existence of any isometry in the curved background. Different types of relativistic quantum type operations $L \longrightarrow L^{-1}$ appear in string theory due to the existence of the dimensional string constant α' , linking different equivalent string theories (the most known is T-duality). refs [12], [13]. The duality here we are considering is of the type classical-quantum (or wave-particle) duality relating classical/semiclassical and quantum behaviours or regimes, during the *same physical process*. For example, the process of black hole evaporation or that of cosmological evolution, refs [16],[17].

Here we present evidence for classical-quantum \mathcal{R} -dual relations, without requiring any symmetry, isometry or compactified dimensions. This evidence is supported by explicit dynamical computations of QFT and string dynamics in curved backgrounds and we illustrate it with two relevant examples: black holes and de Sitter space-time. (AdS states can be included, an explicit discussion is given in ref [17]).

This \mathcal{R} - transformation being of the type of a classical-quantum duality relationship, we do not attach a priori any symmetry to it. Is known the role played by (global or asymptotic) isometries in QFT (and strings) on curved backgrounds, for instance for identifying the particle oscillatory modes, the presence of event horizons, global or asymptotic thermality, etc; however such symmetries are *not* responsible of the classical-quantum duality.

An enormous amount of work was devoted recently to the holographic AdS/CFT correspondence refs [14], [15]. Our work does not make use of the AdS/CFT conjecture.

QFT in curved backgrounds with event horizons possesses an intrinsic (Hawking-Gibbons) temperature T_H Ref. [1,10,11], which can be expressed, in general, as a function T of L_{cl} (and of the constants \hbar , c and k_B)

$$T_H = T(L_{cl}) \quad (4)$$

Quantum strings in Minkowski space time have an intrinsic (Hagedorn) temperature. Quantum strings in curved backgrounds have also an intrinsic string temperature T_S Ref. [2,3,6], which depends on L_q

$$T_S = T(L_q) \quad (5)$$

Explicit calculations for de Sitter (dS) and Schwarzschild Black Hole (BH) show that T is formally the same function for both QFT and QST temperatures ([Eq. (4)] and [Eq. (5)]), Ref. [3,4].

It is worth to point out that space times without event horizons – such as anti de Sitter (AdS) space – have neither T_H nor T_S temperatures, independently of how much quantum matter is present. In fact, in pure AdS space, T_H is zero and T_S is formally infinite Ref. [6,9].

Applying the \mathcal{R} operation ([Eq. (2)] to [Eq. (4)] and [Eq. (5)], we read

$$\tilde{T}_H = T_S, \tilde{T}_S = T_H \quad (6)$$

That is, T_H and T_S are \mathcal{R} -mapped one into the other. From the above equations, we can also write

$$\tilde{T}_H \tilde{T}_S = T_S T_H \quad (7)$$

which is a weaker \mathcal{R} -relation between T_H and T_S .

Let us analyse the mass domains in the corresponding QFT and QS regimes in a curved space time. They will be limited by the corresponding mass scales M_H and M_{QS} . M_H depends on the classical length L_{cl}

$$M_H = M(L_{cl}) \quad (8)$$

and M_{QS} depends on the quantum length L_q , through the same formal relation. Refs [16], [17]

$$M_{QS} = M(L_q) \quad (9)$$

Under the \mathcal{R} operation [Eq. (2)], the scales of mass satisfy:

$$\tilde{M}_H = M_{QS}, \tilde{M}_{QS} = M_H \quad (10)$$

On the other hand, if m_{QFT} is the mass of a test particle in the QFT regime and m_S the mass of a particle state in the quantum string spectrum, the mapping of the QFT mass domain \mathcal{D} onto the QS mass domain – and viceversa – reads

$$\mathcal{R}(\mathcal{D}(m_{QFT}, M_H)) = \mathcal{D}(m_S, M_{QS}) \quad (11)$$

(See refs [16], [17] for more details). Summarizing: The QFT regime, characterized by $(L_{cl}, T_H$ and $M_H)$, and the QS regime – characterized by $(L_q, T_S$ and $M_{QS})$ –, both in a curved space background, are mapped one into another under the \mathcal{R} -transform. The set (L_q, T_S, M_{QS}) is the quantum string dual of the classical/semiclassical QFT set (L_{cl}, T_H, M_H) . Quantum gravity regime and classical/ semiclassical gravity regime are \mathcal{R} dual of each other.

We illustrate this duality relation and its meaning with two relevant examples: de Sitter (dS) and Black Hole (BH) space times.

3 QFT and QST in de Sitter space time

The classical L_{cl} , or horizon radius, is

$$L_{cl} = cH^{-1} \quad (12)$$

where H is the Hubble constant. The mass scale M_H is such that L_{cl} is its Compton wave length

$$M_H = \frac{\hbar}{cL_{cl}} = \frac{\hbar H}{c^2} \quad , \quad (13)$$

and the QFT Hawking-Gibbons temperature is

$$T_H = \frac{\hbar}{2\pi k_B c} \kappa \quad (14)$$

here κ is the surface gravity. For dS space time $\kappa = cH$, and T_H reads

$$T_H = \frac{\hbar H}{2\pi k_B} = \frac{\hbar c}{2\pi k_B} \left(\frac{1}{L_{cl}} \right) \quad (15)$$

On the other hand, canonical as well as semiclassical quantization of quantum strings in dS space time Ref. [6,7,8,] lead to the existence of a maximum mass $m_{\max} \sim c(\alpha' H)^{-1}$ for the quantum string (oscillating or stable) particle spectrum. This maximal mass identifies the mass scale M_{QS} in the string regime:

$$M_{QS} \equiv m_{\max} \simeq c(\alpha' H)^{-1} \quad (16)$$

The fact that there is a maximal mass M_{QS} implies the existence of a (minimal) quantum string scale L_q , which is the corresponding Compton wave length

$$L_q = \frac{\alpha' \hbar H}{c^2} = \frac{\hbar}{cM_{QS}} \quad , \quad (17)$$

and of a maximum (or critical) temperature T_S ,

$$T_S = \frac{\hbar c}{2\pi k_B} \left(\frac{1}{L_q} \right) = \frac{c^3}{2\pi k_B \alpha' H} \quad (18)$$

This temperature is the string temperature emerging from the asymptotic (highly excited) mass spectrum of strings in de Sitter background. This temperature also appears to be the intrinsic temperature of de Sitter stage in its quantum gravity (string) regime,

refs [3], [17]. (As is known, there is no de Sitter string conformal invariant background from the low effective string eqs, see refs [3], [17]).

L_q , T_S and M_{QS} depend on the string tension and on H , while $\mathcal{L}_{\mathcal{R}}$ [Eq. (3)] does only on α' :

$$\mathcal{L}_{\mathcal{R}} = \alpha' \hbar c^{-1} \equiv L_S^2 \quad (19)$$

L_S is a pure string scale.

The \mathcal{R} -transform maps the semiclassical/ QFT set (L_{cl} , T_H and M_H) into the quantum string set (L_q , T_S and M_{QS}), satisfying the duality relations [Eqs. (2,6 and 10)].

If back reaction effect of the quantum matter is considered, \mathcal{R} -relations between the QFT and QST regimes manifest as well, ref [3]. We studied quantum string back reaction due to the higher excited modes in the framework of the string analogue model. Two branches R_{\pm} (i.e. H_{\pm}) of solutions for the scalar curvature show up ref. [3]:

- (a) A high curvature solution R_+ with a maximal value $R_{\max} = (9c^4\pi^2/4G)(6/(5\alpha'c\hbar^3))^{1/2}$, entirely sustained by the strings. This branch corresponds to the string phase for the background, whose temperature is given by the *intrinsic string de Sitter temperature*

$$T_S \equiv T_+ = \frac{c^3}{2\pi k_B \alpha' H_+} \quad . \quad (20)$$

- (b) A low curvature solution R_- whose leading term in R/R_{\max} expansion is the classical curvature. This branch corresponds to the semi-classical-QFT phase of the background, whose temperature is given by the *intrinsic QFT de Sitter (Hawking-Gibbons) temperature*

$$T_H \equiv T_- = \frac{\hbar H_-}{2\pi k_B} \quad (21)$$

Then, the dual classical-quantum \mathcal{R} -relations manifest as well when *back reaction is included*: the stringy regime (a) at the string temperature T_S eq (21) and the semiclassical phase (b) at the QFT Hawking-Gibbons temperature T_H eq (22) are the quantum-classical \mathcal{R} -dual of each other. The branches (R_+ , T_+) and (R_- , T_-) are the \mathcal{R} -transformed of each another.

4 QFT and QST in Schwarzschild BH space time

Here L_{cl} is the BH radius r_H (G : gravitational Newton constant):

$$r_H = \left(\frac{16\pi G M_H}{c^2(D-2)A_{D-2}} \right)^{\frac{1}{D-3}}, \quad \left(A_{D-2} \equiv \frac{2\pi^{\frac{(D-1)}{2}}}{\Gamma\left(\frac{D-1}{2}\right)} \right) \quad (22)$$

and M_H is the BH mass. The QFT Hawking temperature is

$$T_H = \frac{\hbar c(D-3)}{4\pi k_B} \left(\frac{1}{r_H} \right) \quad (23)$$

($\kappa = (D-3)c^2/2r_H$). In string theory, BH emission is described by an incomplete gamma function of $(T_S - T_H)$ Ref. [4]. In the QFT regime, the BH does emit thermal radiation at a temperature T_H Ref. [10]. In the QST regime, the BH has a high massive string emission, corresponding to the higher excited quantum string states Ref. [4]. For open strings (in the asymptotically flat BH region), the thermodynamical behaviour of these states is deduced from the string canonical partition function Ref. [4]. In BH backgrounds, the mass spectrum of quantum string states coincides with the one in Minkowski space, and critical dimensions are the same as well, Ref. [5]. Therefore, the asymptotic string mass density of levels, in BH space times, reads $\rho(m) \sim \exp\{b(\alpha'c/\hbar)^{1/2}m\}$, and quantum strings have an intrinsic temperature T_S which is the same as in flat space time. The string canonical partition function is defined for Hawking temperatures T_H satisfying the condition Ref. [4]

$$T_H < T_S = \frac{\hbar c}{bk_B L_S} \quad (24)$$

i.e in string theory, T_H has an upper limit given by the intrinsic or critical string temperature T_S . This limit implies the existence of a minimal BH radius r_{\min} , and a minimal BH mass M_{\min}

$$r_H > r_{\min} = \frac{b(D-3)}{4\pi} L_S \quad (25)$$

and

$$M_H > M_{\min} = \frac{c^2(D-2)A_{D-2}}{16\pi G} \left(\frac{b(D-3)L_S}{4\pi} \right)^{D-3} \quad (26)$$

Here L_S is given by [Eq. (19)], and \mathcal{L}_R [Eqs.(2,3)] is given by

$$\mathcal{L}_R = \frac{bM_H\alpha'^{\frac{3}{2}}}{2\pi} \sqrt{\frac{\hbar}{c}}, \quad D = 4.$$

Therefore, the QS scales are $L_q = r_{\min}$, $M_{QS} = M_{\min}$, and the string temperature T_S . They depend on the type of strings and on the dimension through the parameter b . In terms of r_{\min} , T_S and M_{\min} read

$$T_S = \frac{\hbar c(D-3)}{4\pi k_B} \left(\frac{1}{r_{\min}} \right) \quad (27)$$

$$M_{\min} = \frac{c^2(D-2)A_{D-2}}{16\pi G} r_{\min}^{D-3} \quad (28)$$

The set (r_H, M_H, T_H) Eqs.(22)-(23) and the set $(r_{\min}, M_{\min}, T_S)$ Eqs.(24)-(26) are \mathcal{R} -transformed of each other: the set $(r_{\min}, M_{\min}, T_S)$ is the quantum dual of the semiclassical set (r_H, M_H, T_H) . This is valid in all dimensions.

The physical meaning (shown by the explicit results of Ref.4) behind these classical-quantum \mathcal{R} -relations is the following. At the first stages of BH evaporation, emission

is in the lighter particle masses at the Hawking temperature T_H , as described by the semiclassical - QFT regime. As evaporation proceeds, temperature increases and high massive emission corresponds to the higher excited string modes. At the later stages, for $T_H \rightarrow T_S$ (i.e $r_H \rightarrow r_{\min}$, $M_H \rightarrow M_{\min}$), the BH enters its QS regime. The \mathcal{R} transformation allows to link the early or semiclassical ($T_H \ll T_S$, i.e $r_H \gg r_{\min}$, $M_H \gg M_{\min}$) and the late or stringy ($T_H \rightarrow T_S$, i.e $r_H \rightarrow r_{\min}$, $M_H \rightarrow M_{\min}$) stages of BH evaporation.

These classical-quantum \mathcal{R} -relations manifest as well if the *back reaction effect of higher massive string modes is included* Ref. [4]: The string back reaction solution (r_+ , M_+ , T_+) shows that the BH radius r_+ and mass M_+ decrease and the BH temperature T_+ increases. Here r_+ is bounded from below (by r_{\min}) and T_+ *does not blow up* (T_S is the maximal value). The string back reaction effect is finite and consistently describes both the QFT ($T_H \ll T_S$) and the QST ($T_H \rightarrow T_S$) regimes. It has the bounds: $(r, T, M)_{\min} < (r, T, M)_+ < (r, T, M)_H$. The two bounds are linked by the \mathcal{R} transform, they are the classical-quantum dual of each other, ie the \mathcal{R} transform links the early (QFT) and the last (QST) stages of evaporation.

5 Conclusions

From the above BH and dS studies, the Hawking-Gibbons temperature of the semiclassical gravity QFT regime becomes the intrinsic string temperature of the quantum string regime.

Also, in the AdS background, the \mathcal{R} -transformation manifests as well between T_H and T_S .

When back reaction is included, the \mathcal{R} -dual relations between semiclassical (QFT) and stringy phases manifest as well.

These BH and dS examples suggest that our \mathcal{R} transform could be promoted to a dynamical operation: evolution from a semiclassical gravity - QFT phase to a quantum gravity string phase (as in BH evaporation) or conversely, evolution from a quantum string phase to a semiclassical QFT phase (inflation) (as in cosmological evolution). These issues, as well as other unifying concepts between black holes and elementary particles, have been further developed and elaborated in Ref 17, including the corresponding density of states and entropies in the both regimes, and the quantum dS and AdS string states. The last BH string state decays with a decay rate which is precisely the \mathcal{R} transformed of the semiclassical thermal BH decay formula,

A particular consequence of these results is that there is no lost of information in black hole evaporation (there is no paradox at all). The results by these authors on the last stage of black hole evaporation, its emission spectrum and its final decay in pure radiation, are reported in Ref [4] and in Ref [17]. Recently, Stephen Hawking, Ref [18] presented, within another approach (euclidean QFT gravity), his resolution of the “information paradox”, a problem which himself have clearly posed and pioneered. These are very important news for the subject, and encourage new roads to the understanding of quantum black holes, very early cosmology and the classical-quantum dual nature of Nature .

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